

## Ionospheric specification algorithms for precise GPS-based aircraft navigation

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**Abstract.** The Federal Aviation Administration (FAA) is implementing an aircraft navigation scheme for the United States using the Global Positioning System (GPS)  $L_1$  signal (1575.42 MHz). To achieve position accuracies of a few meters, sufficient to allow precision airfield approaches, it will be necessary to broadcast corrections to the direct GPS signal. A significant component of these corrections is the delay in the GPS signal introduced by its propagation through the ionosphere. Ionospheric delay corrections will be derived using a ground network of at least 24 dual-frequency GPS receivers distributed across the continental United States. This network is part of the FAA's wide area augmentation system (WAAS) and will provide real-time total electron content (TEC) measurements. We present a technique for converting these TECs into gridded vertical delay corrections at the GPS  $L_1$  frequency, which will be broadcast to users every 5 min via geosynchronous satellite. Users will convert these delays to slant corrections for their own particular lines of sight to GPS satellites. To preserve user safety, estimates of the error in the user delay corrections will also be broadcast. However, the error algorithm must not resort to excessive safety margins as this reduces the expected accuracy, and thus utility, of the navigation system. Here we describe an error algorithm and its dependence on various factors, such as user location with respect to the WAAS ground network and ionospheric conditions.

### 1. Introduction

The Federal Aviation Administration (FAA) has determined that the Global Positioning System (GPS) can be used as the primary navigation tool for commercial and general aviation in the United States. In addition to way point navigation, such a GPS-based system must provide sufficient accuracy to perform precision approaches to airfields, and must provide this accuracy reliably [RTCA Special Committee 159, 1996].

GPS user position and velocity solutions are produced by combining knowledge of GPS space vehicle (SV) positions with measurements of the travel times of signals from four or more of those SVs. Travel times are obtained from prior knowledge of unique, synchronized pseudorandom codes that are modulated on each SV's signal carrier waves, while the SV positions are encoded in the broadcast signal. For

civilian users the largest uncertainties in the travel time measurements currently come from transmitter clock uncertainties and ionospheric delays.

To mitigate this, the FAA instigated the wide area augmentation system (WAAS), which includes a network of GPS receivers distributed over at least 24 sites in the continental United States (CONUS) [Enge and Van Dierendonck, 1996]. The main purpose of the WAAS is to provide a "differential GPS" service, which can derive and broadcast corrections to reduce the uncertainties in the transmitter clocks, ionospheric delays, and SV positions.

To permit calibration of ionospheric delays, the GPS SVs transmit signals at two frequencies,  $L_1$  (1575.42 MHz) and  $L_2$  (1227.6 MHz). However, the pseudorandom code needed to extract signal travel times, and thus SV ranges and navigation solutions, is only guaranteed to be available to civilian aircraft on  $L_1$ . WAAS will therefore be used to calibrate ionospheric conditions over the CONUS and to broadcast ionospheric delays to civilian aviation users.

Ionospheric calibrations will be calculated from data provided by dual-frequency receivers, installed at

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permanent WAAS reference sites (WRSs), which are able to measure the signal travel times at both frequencies. This receiver network will make ionospheric delay measurements every second, which will be used to update delay correction values at ionospheric grid points (IGPs) across the CONUS every 5 min. These IGP delays will then be transmitted to users via geosynchronous satellite.

Aircraft use these delays by first determining the ionospheric pierce points (PPs) of the propagation paths to SVs in their field of view. They then calculate vertical ionospheric delays for these points by interpolating IGP correction values and apply an obliquity function,  $M(E)$ , to reproduce the slant delays.  $M(E)$  converts between vertical and slant delays and depends on the elevation of the slant path and the ionospheric representation in use.

To allow users to estimate the accuracy of their navigation solutions, information on the accuracy of the ionospheric delay calibrations is provided by the grid ionosphere vertical errors (GIVEs), which are meant to bound the errors in the vertical TEC estimates at the IGP locations. The GIVE values, broadcast along with the calibrations themselves, ensure aircraft safety. However, excessively conservative error estimates would make precision navigation impossible in all but ideal conditions. Ultimately, the GIVE is not only an estimate of vertical TEC errors but also the only mechanism for bounding the errors in the user's ionospheric slant path delay correction [Mannucci *et al.*, 1999]. We will discuss the mechanisms used to bound the calibration uncertainty below, based on general considerations of error sources that occur in the ionospheric modeling. Future work will ground these considerations using the extensive data expected from the WAAS and other GPS networks.

Although ionospheric delays and errors are considered here in the context of aircraft navigation, the WAAS corrections will be broadcast openly. The WAAS will clearly be used as a source of GPS corrections by a wide variety of opportunistic users in scientific and commercial fields outside of aviation [Klobuchar, 1997].

## 2. Ionospheric Delay Calibrations

The techniques used to derive WAAS ionospheric delays have grown directly from ionospheric total electron content (TEC) mapping procedures developed as an ionospheric science and space weather resource and for other calibration purposes, such as spacecraft

tracking and single-frequency ocean altimetry [Wilson *et al.*, 1995; Mannucci *et al.*, 1998]. These techniques start with line-of-sight (LOS) delay/TEC measurements derived from observations made by regional (in the case of the WAAS) or global dual-frequency GPS receiver networks.

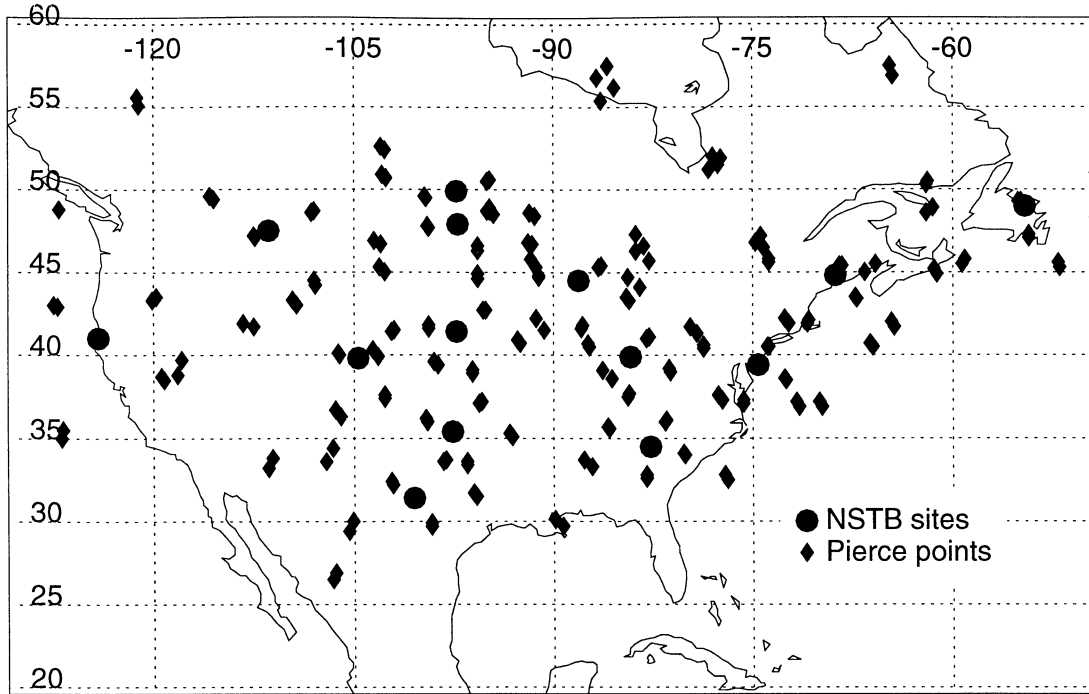
These slant measurements, which include interfrequency biases from the transmitter and receiver hardware, are scaled to vertical using an obliquity function. The latitude and longitude of the resulting vertical delays are defined by where the LOS pierces a reference height associated with the ionosphere. For the purpose of computing ionospheric delays the WAAS specifies the ionospheric model as having all electrons confined to a thin shell at 350 km altitude. For example, ionospheric PPs resulting from 5 min data collected by the FAA's National Satellite Test Bed (NSTB) network mapped to this shell are shown in Figure 1. The NSTB network was being used in place of the WAAS while the WRS receivers underwent testing.

The irregularly distributed WRS delay measurements are incorporated into a dynamic mapping procedure that produces time-varying estimates of the vertical delays at a set of triangular grid points distributed across the Northern Hemisphere and fixed in solar-geomagnetic coordinates [Knecht and Shuman, 1985]. A bilinear interpolation algorithm is used to fit the grid point delays to discrete batches of data, typically of 5 min duration in the case of the WAAS. The delay model is written as follows:

$$I_{rs}(t_i) = M(E) \sum_{i=1}^3 W_i(\lambda_{pp}, \phi_{pp}) V_i, \quad (1)$$

where  $I_{rs}(t_i)$  is the ionospheric delay between receiver  $r$  and SV  $s$  at time  $t_i$ ,  $M(E)$  is the obliquity function,  $W_i(\lambda_{pp}, \phi_{pp})$  is a distance weighting factor which depends on the latitude,  $\lambda_{pp}$ , and longitude,  $\phi_{pp}$ , of the PP within a grid tile, and  $V_i$  is the delay value at triangular grid point  $i$  [Mannucci *et al.*, 1998].

The fit is made using an optimal least squares technique, which solves for all values  $V_i$ . These values are allowed to vary in time as a random walk stochastic process and are updated every 5 min with new measurements. The error covariance matrix for the grid values is also updated. The procedure additionally solves for the relative transmitter and receiver hardware biases and removes them from each pairing of  $r$  and  $s$ . Solutions are generated in a solar-geomagnetic reference frame; the ionosphere typically varies relatively slowly in this frame as most of its driving forces are nominally



**Figure 1.** Ionospheric pierce points for 5 min measurements made by a subset of the NSTB network of receivers.

diurnal or aligned along geomagnetic field lines. Translation to the Earth-fixed WAAS IGPs is performed as the final step [Mannucci *et al.*, 1995]. Estimates made by Raytheon, the WAAS prime contractor, indicate that WAAS requirements can probably be met if the vertical delay corrections are accurate to 60 cm,  $1\sigma$  (this is 3.7 TEC units at the  $L_1$  frequency, where 1 TEC unit (TECU) =  $10^{16}$  el m $^{-2}$ ).

### 3. Ionospheric Delay Calibration Errors

To allow users to calculate the accuracy of the ionospheric delay calibrations, and thus determine their position accuracy, an error value, the GIVE, will be broadcast for each IGP every 5 min [Conker *et al.*, 1997]. In our estimation scheme an error value, based on propagation of errors from the original measurements using the linear model (equation (1)), is available. However, there are several factors that make this unsuitable as the sole source for direct GIVE derivation. Additional considerations are discussed in more detail in sections 3.1-3.4.

#### 3.1. Statistical Error

On the basis of a consideration of measurement noise from the reference sites alone there will be errors

associated with the vertical delay estimates at the grid points  $i$ . Just as a least squares fit of a line to data depends on the number of data points and their spread, the square-root variance of the vertical ionospheric delay estimate  $\sigma_{SE}$  depends on the quantity and spatial distribution of the GPS data. We have found that  $\sigma_{SE}$  increases at the boundaries of the WAAS coverage area, i.e., the CONUS; a desirable feature for ensuring user safety.

However, this error estimate involves several assumptions, such as the level of measurement noise and the approximate linear model relating measurements to the vertical delay grid values  $V_i$ . A quantitative measure of the validity of these assumptions is provided by the  $\chi^2$  ("chi-square") goodness-of-fit metric, which we use to multiply the nominal statistical error,  $\sigma_{SE}$ . The value of  $\chi^2$  increases when the fit is poor, indicating some error in our statistical model of the fitting process. The final expression for the statistical error term is thus

$$\text{GIVE}_{SE} = \alpha(3.29\sigma_{SE})\sqrt{\chi^2}, \quad (2)$$

where  $\alpha$  is a scaling factor used to compensate for systematic underestimates of the formal error, for example, due to overly optimistic data noise. The factor

of 3.29 is introduced as the GIVE is required to bound 99.9% of errors, i.e.,  $3.29\sigma$ . So,  $\chi^2$  is given by

$$\chi^2 = \frac{1}{N} \sum_{i=1}^N \frac{(\text{FIT}_i - \text{DATA}_i)^2}{\sigma_i^2}, \quad (3)$$

where  $\text{DATA}_i$  is a WRS delay measurement with noise  $\sigma_i$  and  $\text{FIT}_i$  is the slant delay at the corresponding measurement PP computed by a user [Bevington, 1969]. The lowest value of  $\chi^2$  used is unity, to prevent underestimation of the statistical error when the data and fit are in good agreement due to happenstance.

$\text{GIVE}_{\text{SE}}$  includes errors from the fitting procedure used to produce the vertical TEC maps but does not reproduce errors incurred as users interpolate the gridded broadcast corrections and map them to slant ray paths. Additional terms are required to reproduce these.

### 3.2. Spatial Decorrelation

The spatial variability, or decorrelation, of the ionosphere becomes important when WRS measurements and user PPs are widely separated [Klobuchar *et al.*, 1995; Chao *et al.*, 1996]. We therefore calculate an additional interpolation-based GIVE term that depends on the distance between the user PP and the WRS measurements. The locations of user PPs are unknown during GIVE generation, so instead we compute the “worst case” distance,  $D_{\text{MAX}}$ , for a user in the region surrounding an IGP. The decorrelation GIVE error term is a simple linear form that increases with this distance:

$$\text{GIVE}_{\text{DEC}} = \beta V_{\text{MAX}} D_{\text{MAX}}, \quad (4)$$

where  $V_{\text{MAX}}$  is the largest vertical ionospheric delay in the four quadrants surrounding the IGP (these quadrants are defined as rectangles having one vertex at the IGP and the other vertices at the locations of the adjacent IGPs) and  $\beta$  is a scaling factor based on estimates of decorrelation derived empirically from experience of our

**Table 1.** GIVE Parameter Values

GIVE Parameter	Value	Comments
$\alpha$	1.4	>1 as errors are non-Gaussian
$\beta$	$1.0 \times 10^{-7} \text{ m}^{-1}$	increases $\text{GIVE}_{\text{DEC}}$ by 5% at 500 km
$\gamma$	$3.0 \times 10^5 \text{ m}$	increases $\text{GIVE}_{\text{GRAD}}$ by 1.5 m (9.25 TEC units) when gradient is $0.5 \text{ cm km}^{-1}$ (maximum daytime value)

**Table 2.** Dates and Ionospheric-Geomagnetic Conditions Analyzed

Activity Level	$A_p$	Date
Quiet	9	August 24-25, 1998
Moderate	64	August 6, 1998
Severe	144	August 26-27, 1998

TEC maps. The dependency on  $V_{\text{MAX}}$  is included as we expect spatial decorrelation errors to increase with the overall level of the ionosphere.

### 3.3. Gradient

WRS measurements are scaled to vertical, and IGP vertical calibration delays are scaled to user slant paths, using the obliquity function  $M(E)$ . Although  $M(E)$  depends on the elevation of a slant LOS, it is independent of the azimuth. Delay calibration errors for users will therefore arise that depend, in part, on the local horizontal electron density gradients found in the real ionosphere that cause azimuthal delay variation [Chavin, 1996; Prag, 1996]. We therefore introduce a gradient-based GIVE term:

$$\text{GIVE}_{\text{GRAD}} = \gamma V_{\text{MAX}}, \quad (5)$$

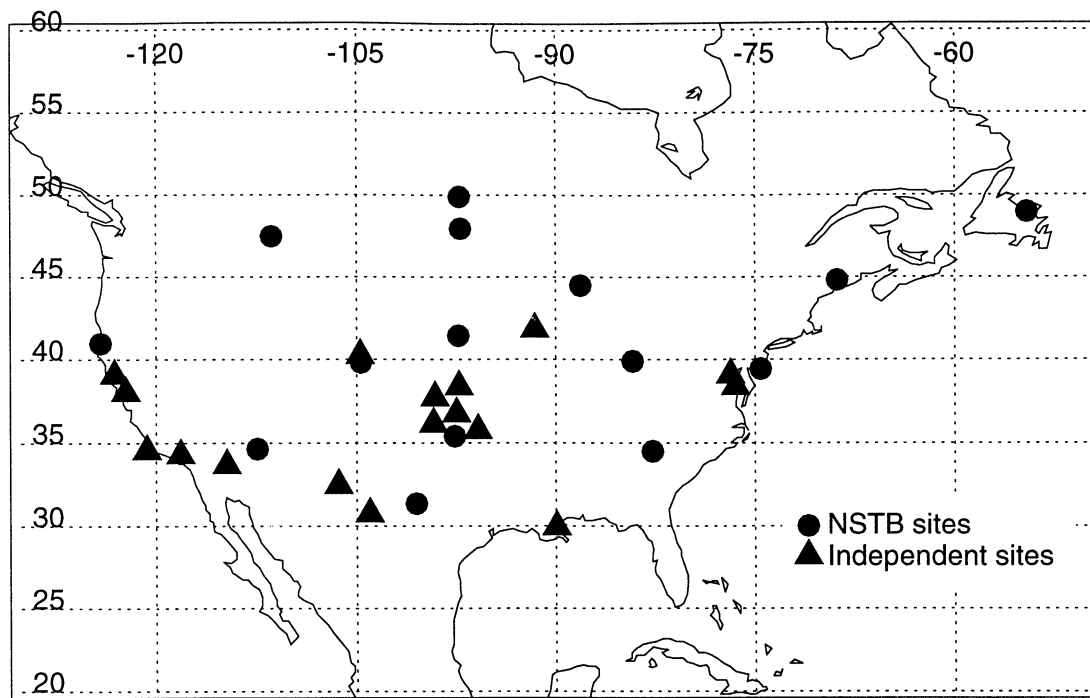
where  $V_{\text{MAX}}$  is an estimate of the maximum delay gradient surrounding the IGP and  $\gamma$  is an empirically tuned scaling factor based on our studies of TEC gradients and obliquity function errors. Note that the gradient estimate is derived from the structure of the vertical delay map and thus shares its limitations regarding the capture of small-scale spatial structures.

### 3.4. Combined GIVE

Scaling factor values used here are given in Table 1. The individual GIVE terms are combined in root-sum-square fashion to form a total,  $\text{GIVE}_{\text{TOT}}$ . For aircraft navigation this is rounded up to one of a discrete set of values available for transmission by the WAAS:

$$\text{GIVE}_{\text{TOT}} = \sqrt{\text{GIVE}_{\text{SE}}^2 + \text{GIVE}_{\text{DEC}}^2 + \text{GIVE}_{\text{GRAD}}^2}. \quad (6)$$

This method of combining GIVE terms is appropriate when the different error contributions are statistically uncorrelated. Although this is not strictly

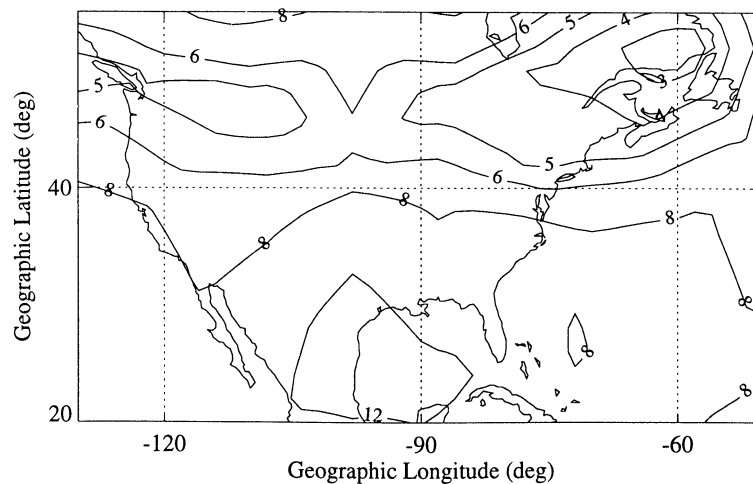


**Figure 2.** Map showing the locations of NSTB and independent GPS receivers used in this study.

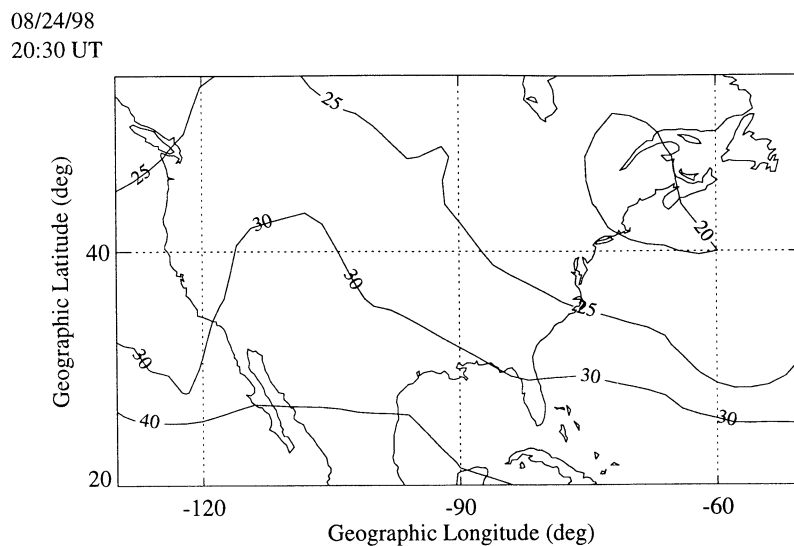
true here, we have found that a direct summing of the different terms tends to produce an overly conservative GIVE. To provide a precision approach service under the current WAAS requirements, the  $GIVE_{TOT}$  values

must be less than 2 m (12.2 TECU) a substantial fraction of the time (>90%). However, to ensure safety, they must also bound the true user calibration errors more than 99.9% of the time [Ahmadi et al., 1997].

08/24/98  
08:30 UT



**Figure 3.** Ionospheric delay correction map for a quiet ionosphere nighttime case. Contours are in TECU.



**Figure 4.** Ionospheric delay correction map for a quiet ionosphere daytime case. Contours are in TECU.

#### 4. Preliminary Algorithm Testing

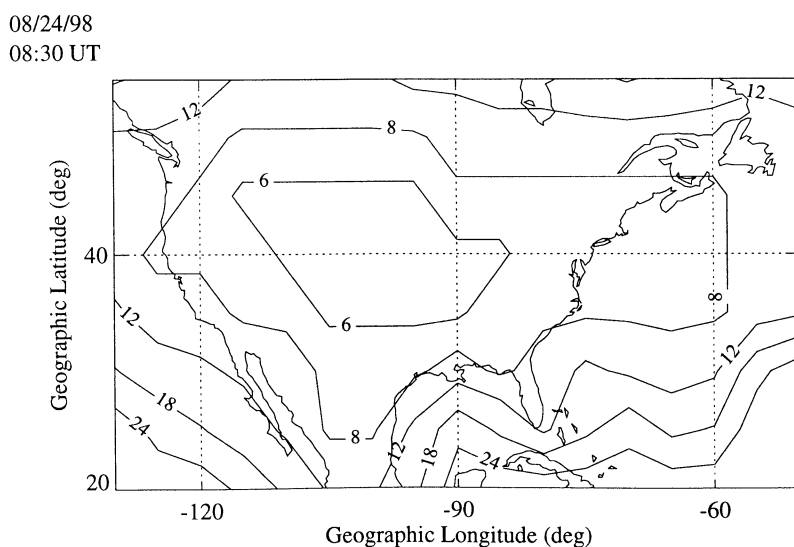
Ionospheric delay calculations and errors are illustrated here using data captured in real time from a subset of the NSTB network, while a comparison is provided by an independent network of receivers across the CONUS (Figure 2). Data from the latter additionally benefit from postprocessing.

Results are shown for three different levels of ionospheric and geomagnetic activity in August 1998 (Table 2). Note that although high  $A_p$  indices

correspond with disturbed ionospheric conditions in these examples, this is not always the case.

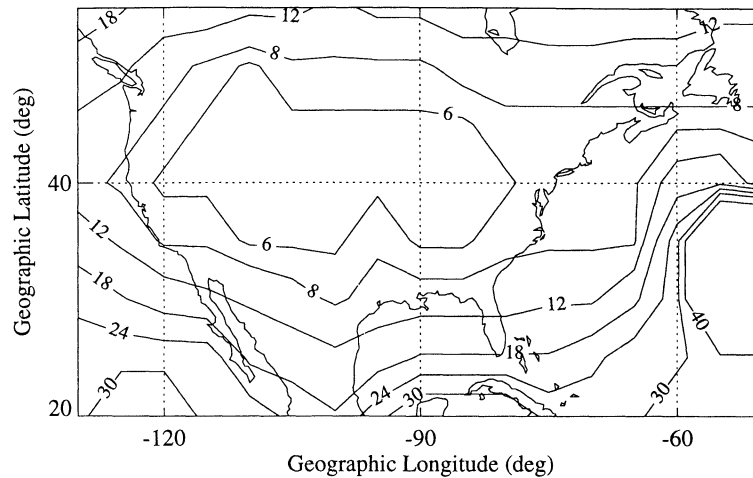
##### 4.1. Quiet Conditions

Figures 3 and 4 show snapshots of ionospheric conditions on August 24, 1998, a “quiet ionosphere” day (contours are marked in TECU). As expected, TECs and delays are greater during the daytime than during the night and generally increase toward the south. The GIVEs are also slightly higher during the



**Figure 5.** GIVE map for the quiet ionosphere nighttime case. Contours are in TECU.

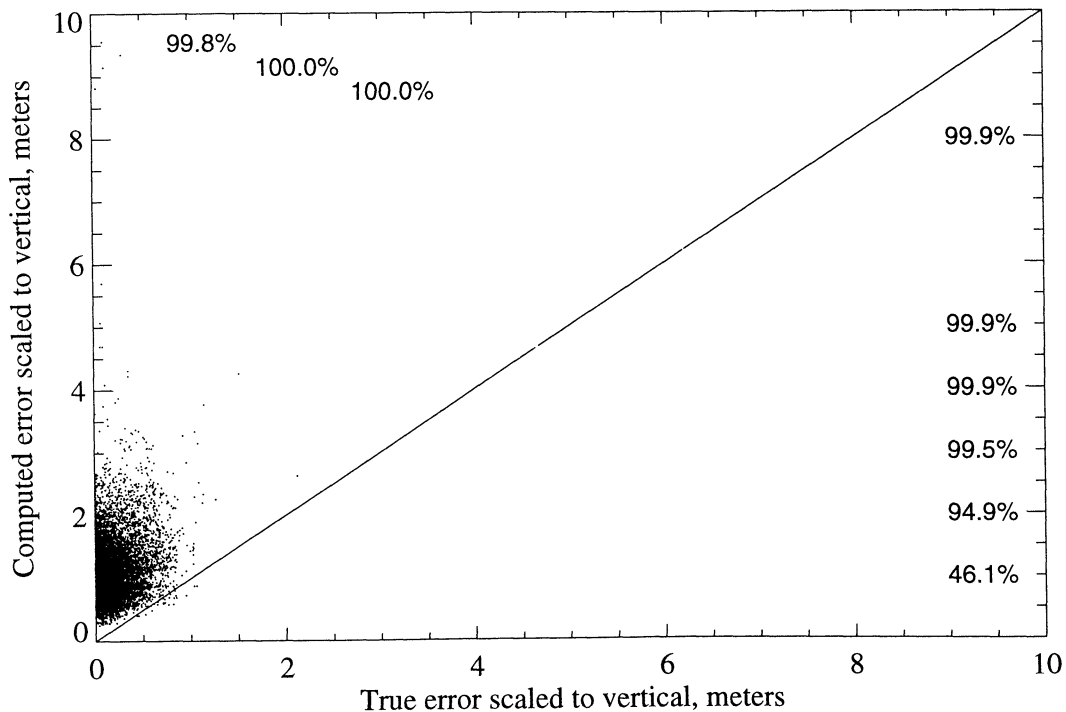
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**Figure 6.** GIVE map for the quiet ionosphere daytime case. Contours are in TECU.

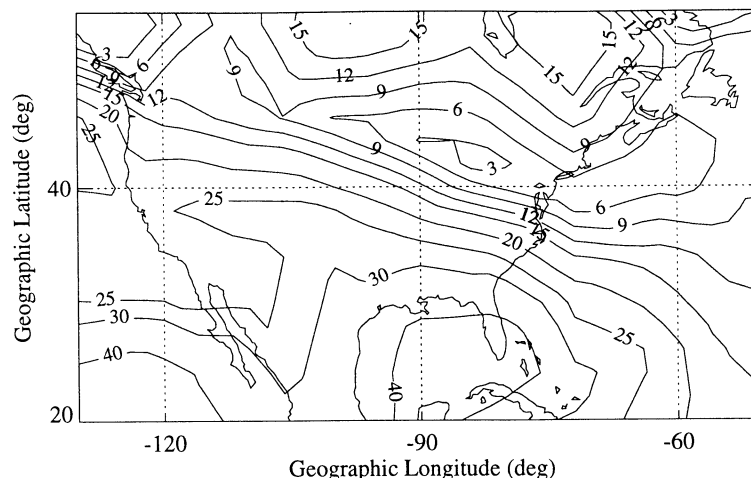
daytime and usually increase toward the boundaries of the WAAS coverage, i.e., the CONUS (Figures 5 and 6). Outside the covered region the GIVEs can increase rapidly, for example, in the region centered approximately on 30°N, 55°E in Figure 6.

Figure 7 shows how the optimization of the GIVE, between conservatively estimated errors and the desire to improve precision, is resolved for this quiet ionosphere case. The figure shows computed errors based on GIVE for the independent network, compared



**Figure 7.** GIVE validation results for quiet ionospheric conditions.

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02:30 UT



**Figure 8.** Ionospheric delay correction map for severely disturbed conditions. Contours are in TECU.

to the corresponding “true” errors, where the latter are the differences between the delay corrections and the independent data. Points above the  $x = y$  line indicate that the GIVE has successfully bounded the true error. Points well above the  $x = y$  line indicate GIVEs that were too conservative.

In this case the GIVE failed to bound the true error on 27 occasions out of 10,148, a high degree of reliability. Here 94.9% of the GIVEs were  $\leq 2$  m, which is not considered overly conservative.

#### 4.2. Moderate Conditions

August 6, 1998, was chosen as an example of a moderately disturbed ionosphere, a condition that arises roughly every two weeks in northern midlatitudes. Delay corrections and GIVE terms derived for this day were similar to those for quiet conditions. This time the GIVE failed to bound the true error in 7 cases out of 13,495, while 93.8% of the GIVEs were  $\leq 2$  m.

#### 4.3. Severe Conditions

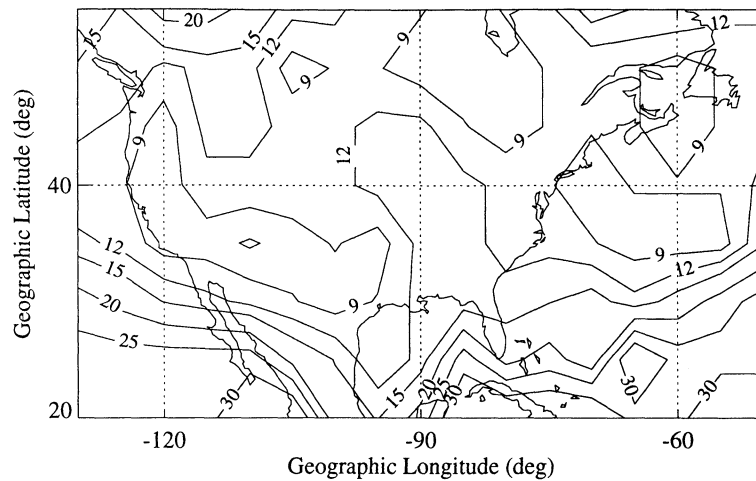
An intense geomagnetic storm occurred on August 26–27, 1998, where the  $A_p$  index reached over 140. Such storms will affect the ionosphere several times a year, and snapshots of conditions and GIVEs for this example are displayed in Figures 8 and 9. The main electron enhancement occurred late on August 26 and early on August 27, so plots are shown for the evening of August 26, local time.

Delay corrections and GIVEs are slightly higher, with sharper gradients, over most of the CONUS than for the corresponding time of day in the quiet ionosphere case. Note in particular the narrow region of electron depletion, or trough, centered at about 43°N, 85°E. The most significant feature for GPS-based aircraft navigation is the increase in incidents of error underestimation by the GIVE: 63 out of 13,197 measurements (0.5%). The largest discrepancy was 0.9 m (Figure 10). A second feature is the increase in the average GIVE; 10.7% were  $\geq 2$  m, while the true error remained below 2 m in 99.6% of cases. This indicates that slightly more conservative GIVEs were produced than for the quiet and moderately disturbed ionosphere cases, despite the increase in underestimates. This would decrease the availability of precision approach coverage.

Figure 11 shows the contributions of the three GIVE terms to  $GIVE_{TOT}$  at IGP 97 (40°N, 85°E) for a 24 hour period covering the severe storm case. It also plots values at the same location and time of day for the quiet ionosphere case. IGP 97 was chosen as it is close to the center of the ionospheric trough in Figure 8.

Figure 11a shows that the  $GIVE_{SE}$  term increased during the severe storm case, between approximately 2300 UT on August 26 and 0400 UT on August 27. This increase was followed by a period of low values, compared to the quiet ionosphere case. Inspection of the delay maps indicates that these times correspond to the

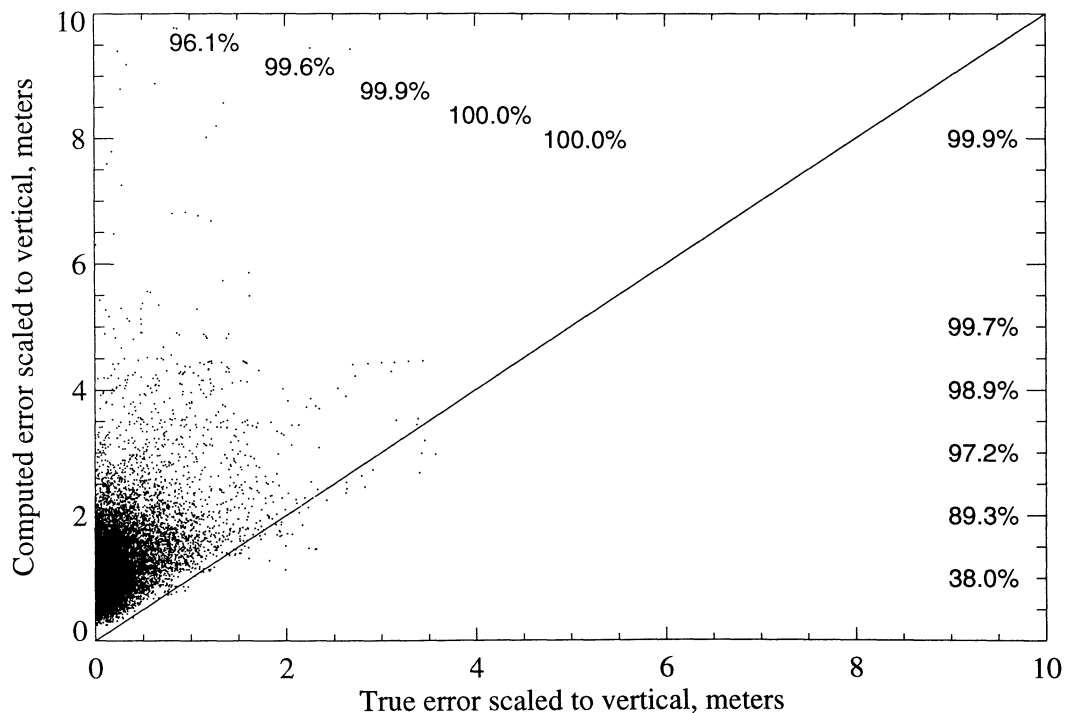
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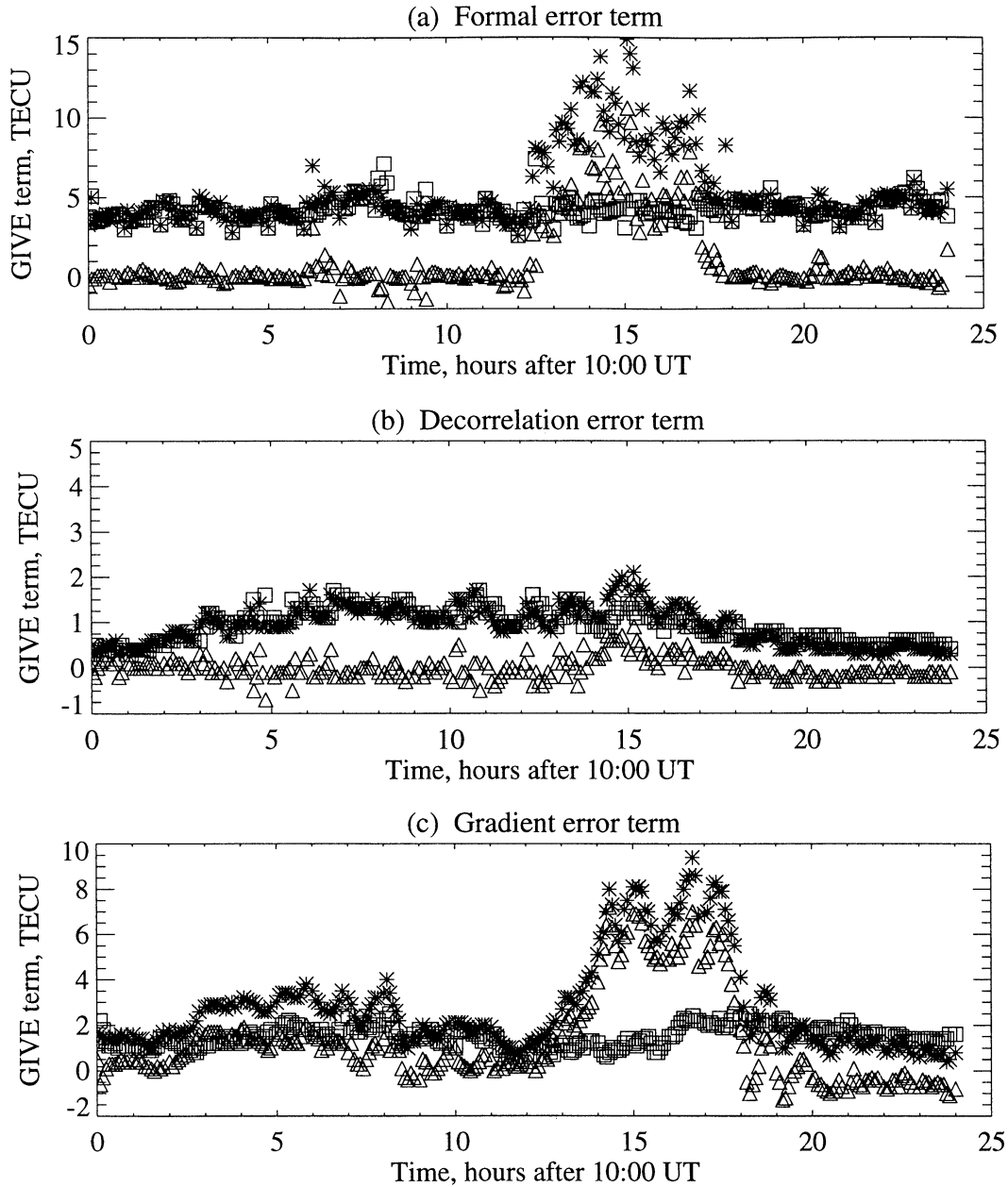
**Figure 9.** GIVE map for severely disturbed ionospheric conditions. Contours are in TECU.

electron enhancement and depletion phases of the storm over the CONUS, respectively. At all times the magnitude of the  $GIVE_{SE}$  term was significantly higher than that of the  $GIVE_{DEC}$  and  $GIVE_{GRAD}$  terms (Figures 11b and 11c, respectively). However, the  $GIVE_{DEC}$  and

$GIVE_{GRAD}$  terms demonstrated an analogous increase and decrease during the severe storm case. Comparisons of GIVE terms at several IGP's suggest that the statistical error term becomes even more dominant close to the edge of the WAAS coverage.



**Figure 10.** GIVE validation results for severely disturbed ionospheric conditions.



**Figure 11.** Time evolution of the (a)  $GIVE_{SE}$ , (b)  $GIVE_{DEC}$ , and (c)  $GIVE_{GRAD}$  terms during quiet (triangles) and severely disturbed (stars) ionospheric conditions. Values for the (disturbed-quiet) difference are also shown (squares). The “quiet” values are for the 24 hour period from 1000 UT on August 24, 1998, while the “disturbed” values are for 24 hours from 1000 UT on August 26, 1998.

Both the  $GIVE_{DEC}$  and  $GIVE_{GRAD}$  terms exhibit diurnal signatures, which could be caused by the daily variation of ionospheric TEC. TEC levels may therefore be partly responsible for the storm-related changes for these terms, for example, by increasing  $V_{MAX}$  and  $\nabla_{MAX}$

in (4) and (5), respectively. Some evidence for this is found by comparing TEC maps for the two ionospheric cases. At 0230 UT on August 24, ionospheric TEC levels and gradients near IGP 97 are lower than for 0230 UT on August 27 (Figure 8), while

values and gradients at 0700 UT on August 24 are similar to or higher than those for the corresponding time on the storm day.

Figure 11b demonstrates that the algorithm does not significantly increase the ionospheric decorrelation rate in the presence of the severe storm trough. This would seem reasonable, but in the current design,  $GIVE_{DEC}$  is not explicitly a function of the TEC gradients. This term is meant to model the residual ionospheric decorrelation that is poorly modeled in the TEC mapping process. Under quiet to moderate conditions we expect these errors to be relatively insensitive to the gradient. However, this approach may not be adequate during disturbed conditions. Further analysis is necessary to refine our model of spatial decorrelation, particularly in the presence of troughs. Substituting the TEC gradient for maximum TEC in (4) may be sufficient.

The electron enhancements and depletions caused by severe ionospheric storms vary in size and location across the midlatitudes, but the trough depicted here can be considered typical of the features that WAAS must contend with. If the increases in the GIVE terms approximately correspond to the passage of enhanced ionospheric gradients across the CONUS, aircraft LOSs transiting trough regions may measure delays significantly lower than the broadcast WAAS corrections. This example demonstrates that algorithm optimization for extreme ionospheric conditions must continue, and we are investigating specialized methods of detecting the onset of extreme ionospheric features so that measures can be employed to further increase the GIVE where necessary.

## 5. Conclusions

A new wide area ionospheric correction technique has been applied to the FAA's WAAS delay correction scheme. Using the CONUS NSTB receiver network, verified with data from an independent network, we have observed that ionospheric delay corrections and gradients increase toward the south, under quiet and moderately disturbed ionospheric conditions. Delay error values can increase with the delay, but they are also dependent on network coverage.

An algorithm to produce the GIVEs, the values that bound the calibration errors, has been derived using three terms: (1) a Kalman-filter-based statistical error derived solely from the measurement system; (2) an error due to the spatial decorrelation of the ionosphere, i.e., spatial interpolation errors; and (3) errors from

converting the vertical corrections to slant paths, i.e., mapping errors.

The main advantage of this algorithm is that the effect of ionospheric decorrelation is considered in the GIVE calculation, in addition to measurement noise. This is an alternative to techniques that use overly restrictive monitoring rules for determining IGP correction availability. For example, *Conker et al.* [1997] apply the requirement of at least one PP in three of the four quadrants adjacent to the IGP. Their analysis produces GIVEs that effectively bound the delay corrections but restricts availability of IGPs at the edges of the CONUS to <96% of the day.

Preliminary testing shows that the GIVE described here successfully bounds users' errors in the great majority (>99.7%) of cases under quiet and moderately disturbed ionospheric conditions. Under severely disturbed conditions the GIVE bounds user errors in most (>99.5%) cases. However, in the latter case there were more incidences of significantly overestimated GIVEs, which would reduce the availability of the precision navigation capability that the WAAS is intended to provide. GIVE underbounding the errors does not necessarily cause hazardous conditions for users as enough safety factors are built in to the user algorithms that derive position accuracy from transmitted error bounds. This important issue will be exhaustively verified in cooperation with the prime WAAS contractor using extensive data sets from a wide variety of ionospheric conditions.

Preliminary analysis indicates that differences between the quiet and severely disturbed performances are due to ionospheric enhancements and depletions and features that are smaller than the spatial scale of the WAAS correction grid. A more detailed investigation of the delay correction calibration and the GIVE terms is therefore warranted, particularly during storm conditions. Daily monitoring of GIVE performance is planned, to tune algorithm parameters and assess performance under a wider variety of conditions.

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